Title	:	A New Definition of Fractional Derivative
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Abstract

In this work, a new definition of fractional derivative and fractional integral are proposed. The authors obtain many useful theorems, and they also give some applications to fractional differential equations.

Keywords: Conformable, Fractional derivative, Fractional integral

Introduction

Fractional derivative is as old as calculus. The most popular of fractional derivative are Riemann-Liourille and Caputo [1-4]. In this work , the authors propose a new definition of fractional derivative and fractional integral as follows.

Definition 1. *Given a function $f : [0,\infty) \to \mathbb{R}$. Then the "conformable fractional derivative" of f of order α is defined by*

$$T_\alpha(f)(t) = \lim_{\varepsilon\to 0} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \quad \forall t > 0, \alpha \in (0,1).$$

If f is α - differentiable in the interval $(0,a)$ for some $a > 0$ and $\lim_{t\to 0^+} f^{(\alpha)}(t)$ exists, then define $f^{(\alpha)}(0) = \lim_{t\to 0^+} f^{(\alpha)}(t)$.

Definition 2. *Let $\alpha \in (n, n+1]$, and f be an n-differentiable at t, where $t > 0$. Then the conformable fractional derivative of f of order α is defined as*

$$T_\alpha(f)(t) = \lim_{\varepsilon\to 0} \frac{f^{(\lceil\alpha\rceil-1)}(t+\varepsilon t^{\lceil\alpha\rceil-\alpha}) - f^{(\lceil\alpha\rceil-1)}(t)}{\varepsilon},$$

where $\lceil\alpha\rceil$ is the smallest integer greater than or equal to α.

Definition 3. *The α - fractional integral of a function f starting from $a \geq 0$ is defined by*

$$I_\alpha^a(f)(t) = I_\alpha^a(t^{\alpha-1}f) = \int_a^t \frac{f(x)}{x^{1-\alpha}}dx,$$

where the integral is the usual Riemann improper integral, and $\alpha \in (0,1)$.

Methodology

As a consequence of the above definitions, we obtain the following useful theorem.

Theorem 1. *Let $\alpha \in (0, 1]$ and f, g be α - differentiable at a point $t > 0$. Then*

1. $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g), \forall a, b \in \mathbb{R}$.
2. $T_\alpha(t^p) = pt^{p-\alpha}, \forall p \in \mathbb{R}$.
3. $T_\alpha(\lambda) = 0$, *for all constant function* $f(t) = \lambda$.
4. $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$.
5. $T_\alpha\left(\frac{f}{g}\right) = \frac{gT_\alpha(f) - fT_\alpha(g)}{g^2}, \ g \neq 0$.
6. *If* f *is differentiable, then* $T_\alpha(f)(t) = t^{1-\alpha}\frac{df}{dt}(t)$.

Theorem 2. $T_\alpha I_\alpha^a(f)(t) = f(t)$, *for* $t \geq a$, *where* f *is any continuous function in the domain of* I_α.

Results

The definitions and theorems are applied to solve conformable fractional differential equation as follows.

Example 1. *Solve* $y^{(\frac{1}{2})} + y = x^2 + 2x^{(\frac{3}{2})}$, $y(0) = 0$.
The particular solution is $y(x) = x^2$.

Example 2. *Solve* $y^{(\frac{1}{2})} + \sqrt{x}y = xe^{-x}$.
The general solution is $y(x) = \frac{2}{3}x^{(\frac{3}{2})}e^{-x} + ce^{-x}$.

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